Proceedings of the
International Conference on Mechanical Engineering and Renewable Energy 2015
(ICMERE2015) 26 – 29 November, 2015, Chittagong, Bangladesh

ICMERE2015-PI-029

Heat Generation Effect on Natural Convection Flow in a Rhombic Shape Cavity Containing a Rectangular Block

Raju Chowdhury[1,*], **Salma Parvin**[2], **Md. Abdul Hakim Khan**[3]

[1]Stamford University Bangladesh, Dhaka-1209, Bangladesh.
[2,3]Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh
[1,*]raju_chy_23@yahoo.com , [2]salpar@math.buet.ac.bd, [3]mahkhan@math.buet.ac.bd

Abstract- *Natural convection heat transfer in a rhombic shaped cavity filled with air and containing a rectangular block is investigated numerically. The top wall of the cavity is cold while the bottom wall is heated isothermally, the left and right inclined walls are adiabatic and the rectangular block is maintained at a cold temperature. The main objective of this study is to explore the influence of pertinent parameters such as Rayleigh number (Ra), heat generation parameter (λ), width of the block (Wx) and height of the block (Hy) on the flow and heat transfer performance of the fluid while the Prandtl number (Pr) is considered fixed. The results are obtained using finite element method and clearly indicate that rectangular block plays an important role on fluid flow and heat transfer. It is also observed that the rate of heat transfer decreases with the increasing of heat generation.*

Keywords: Heat generation, Natural convection, Rhombic cavity

1. INTRODUCTION

Natural convection flow and heat transfer in closed cavities become an important topic to the researchers in the last few decades. The study of such phenomena is important due to its relevance to a wide variety of application area in engineering and science. Some of these include in nuclear reactor cores, fire and combustion modelling, electronic chips and semiconductor wafers etc. These applications are well described by Nield and Bejan [1] and Ingham and Pop [2].

There are plentiful studies in the literature regarding natural convection in enclosures. The phenomenon of heat and fluid flow for a configuration of isothermal vertical walls, controlled at different temperatures and with adiabatic horizontal walls are well explained [3]. Most of the previous studies on natural convection in enclosures are related to either side heating or bottom heating [4-6]. Hasnaoui *et al.* [7] investigated natural convection in an enclosure with localized heating from below. Ganzarolli and Milanez [8] conducted a numerical study for steady natural convection in an enclosure heated from below and symmetrically cooled from the sides. Hossain and Wilson studied natural convection flow in a fluid-saturated porous medium enclosed by non-isothermal walls with heat generation [9]. Chen and Chen [10], Chowdhury *et al.* [11] and Rahman *et al.* [12] have been analyzed heat generation effect on natural convection flow in closed cavities.

Although many works have been conducted using rectangular cavities, very limited works have been done using rhombic cavity. Anandalakshmi and Basak [13] analyzed natural convection in rhombic enclosures with isothermal hot side or bottom wall. Moukalled and Darwish [14] have investigated natural convective heat transfer phenomena in a porous rhombic annulus.

Many authors have recently examined heat transfer in cavities with partitions, fins and blocks which controls the convective flow phenomenon. A body can be used as a control element for heat transfer and fluid flow which is analyzed by Varol *et al.* [15]. Amine *et al.* [16] investigated the thermal convection around obstacles with different configurations. Ha et al. [17] tested the different boundary conditions for the inserted body to the enclosure. Chowdhury *et al.* [18] conducted natural convection in porous triangular enclosure with a circular obstacle in presence of heat generation. They found that the presence of the body obstructs the flow and temperature fields.

In this study, natural convection flow and heat transfer in a rhombic cavity containing a rectangular block with heat generation has been analyzed numerically. The enclosure is constantly heated from bottom wall, cold temperature is maintained at top wall and adiabatic left and right inclined wall. This type of boundary conditions has a practical importance especially in cooled ceiling applications. The main objective of the present study is to examine the effect on size of the rectangular block on flow pattern and heat transfer in the cavity. The tests were performed for different heat generation parameter, Rayleigh number and size of the rectangular block.

2. PHYSICAL FORMULATION

Fig. 1 shows the considered problem of a two-dimensional rhombic shaped cavity of height L with the associated boundary conditions. A cold rectangular body of width wx and height hy is inserted to the center of the cavity and is maintained at cold temperature. The bottom surface is heated with a uniform temperatureT_{hot}, the upper surface is maintained at cold temperature T_{cold} and the left and right inclined walls are adiabatic. The gravitional force acts as indicated in the figure.

3. MATHEMATICAL FORMULATION

In the present study, we assume steady, two-dimensional, laminar flow of a viscous incompressible fluid having constant properties. Under Boussinesq approximation the dimensionless equations of mass, momentum and energy are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{3}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + RaPr\theta, \tag{4}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) + \lambda\theta. \tag{5}$$

where Prandtl number $Pr = \frac{\nu}{\alpha}$, Rayleigh number $Ra = \frac{g\beta(T_{hot}-T_{cold})L^3}{\alpha\nu}$ and heat generation parameter. $\lambda = \frac{Q_0 L^2}{\rho\nu C_p}$.

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables:

$$X = \frac{x}{L},\quad Y = \frac{y}{L},\quad P = \frac{p}{\rho U_0^2},\quad H = \frac{l}{L},\quad Wx = \frac{wx}{L},$$
$$Hy = \frac{hy}{L},\quad U = \frac{u}{U_0},\quad V = \frac{v}{U_0},\quad \theta = \frac{T - T_{cold}}{T_{hot} - T_{cold}} \tag{6}$$

where $U_0 = \frac{\alpha}{L}$ is the reference velocity. where X and Y are the coordinates varying along horizontal and vertical axes respectively, U and V are the velocity components along the X and Y directions respectively, θ is the dimensionless temperature and P is the non-dimensional pressure.

The dimensionless initial and boundary conditions are as follows:

All boundaries are rigid and no-slip; $U = V = 0$

On the bottom surface: $\theta = 1$

On the upper surface: $\theta = 0$

On the left and right inclined surface: $\frac{\partial \theta}{\partial n} = 0$

At the rectangular body:

On the boundary: $U = V = 0$; $\theta = 0$.

Local and mean Nusselt numbers of the bottom surface are calculated using Eqn. (7) and (8) respectively,

$$\overline{Nu} = \frac{hL}{k} \tag{7}$$

where, h represents the heat transfer coefficient, k thermal conductivity.

$$\text{and } Nu = \int_0^1 \overline{Nu}\, dX. \tag{8}$$

4. SOLUTION METHODOLOGY

The numerical procedure used in this study is based on the Galerkin weighted residual method of finite element method. In this method, the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations (i.e. mass, momentum and energy equations) are transferred into a system of integral equations by applying Galerkin weighted residual method. The integration involved in each term of these equations is performed by using Gauss's quadrature method. The nonlinear algebraic equations so obtained are modified by imposition of boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using Triangular Factorization method.

5. MODEL VALIDATION

The numerical simulation for pure fluid with the effect of circular obstacle has been investigated by Parvin and Nasrin [19] for $Ra = 10^5, Pr = 0.7$ and $Ha = 50$. A test has made to contrast the results obtained by the current model with earlier study [19]. Fig. 2 represents the result of streamlines and isotherms for free convection in a square cavity with circular heated block. The left column presents the result of Parvin and Nasrin [19] and right column shows the result for current study. The comparison shows that the present's results agree with the numerical solution of Parvin and Nasrin [19].

6. GRID INDEPENDENCY TEST

To establish the proper grid size for this study, a grid independence test is conducted for $\boldsymbol{Pr = 0.7, Ra = 10^5, Wx = 0.2, Hy = 0.2}$ **and** $\boldsymbol{\lambda = 0}$ which are shown in Table 1. The extreme value of average Nusselt number (*Nu*) is used as the sensitivity measure of the accuracy of the solution and selected as the monitoring variable. Considering the correctness of numerical values, the current calculations are performed with 8,348 nodes and 3,842 elements grid system.

Table: 1

Nodes	5092	5712	8348	17016
Elements	2216	2426	3842	7441
Nu	4.41054	4.40511	4.40187	4.40187
Time (s)	120	142	210	312

7. RESULT AND DISCUSSION

The natural convection inside a rhombic obstructed cavity containing a rectangular block with heat generation effect is influenced by the controlling parameters $10^3 \le Ra \le 10^5, 0 \le \lambda \le 10$, width of the rectangle Wx and height of the rectangle Hy. The results are represented in terms of streamlines, isotherms and average Nusselt number for various range of parameters $Ra,\ \lambda,\ Wx$ and Hy while $Pr = 0.71$ are kept fixed.

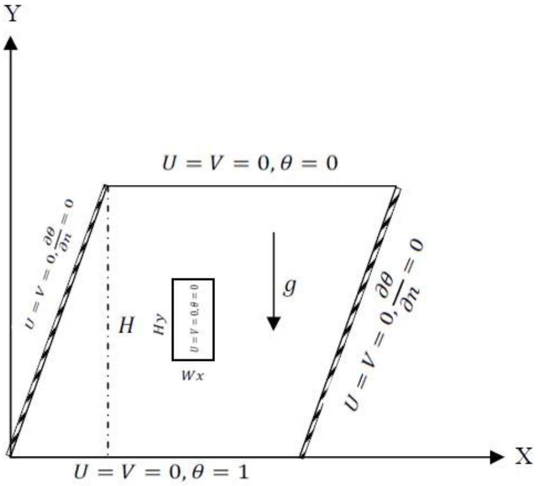

Fig.1 Physical model of the considered problem with corresponding non-dimensional boundary conditions.

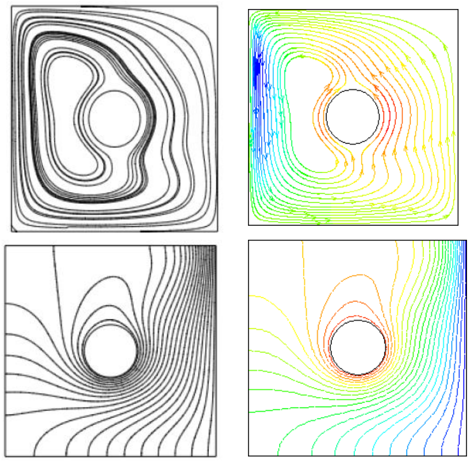

Fig.2. A comparison for streamlines (top row) and isotherms (bottom row) between Parvin and Nasrin [19] (on the left) and present study (on the right) for $Ra = 10^5, Pr = 0.7$ and $Ha = 50$.

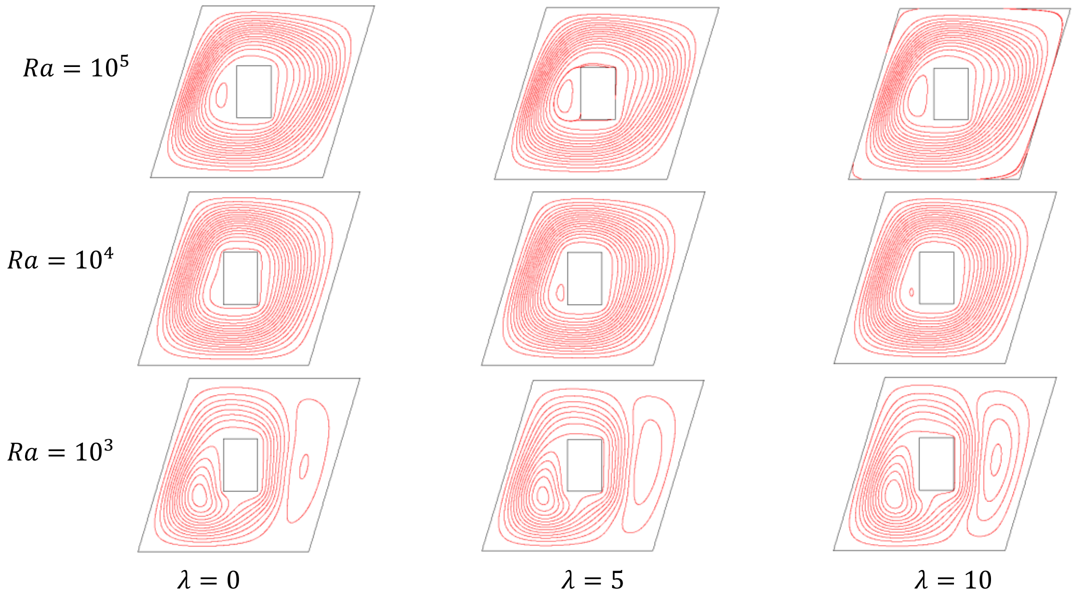

Fig.3. Streamlines for different values of Ra and heat generation parameter λ with $Pr = 0.71, Wx = 0.2$ and $Hy = 0.3$.

Fig. 3 represents streamlines for different values of Ra and heat generation parameter λ while $Pr = 0.71, Wx = 0.2$ and $Hy = 0.3$. It can be shown that two vortices are formed within the cavity where the large vortex near the left inclined wall contains the rectangular block and dominates the other vortex formed near the right inclined wall of the rhombic cavity for $\lambda = 0$ and $Ra = 10^3$. For $Ra = 10^3$, as increasing the value of λ, the size of the minor vortex increases and occupies the space of the cavity due to increasing heat generation while size of the major vortex remains almost same. The minor vortex decreasing and a rhombic shaped flow occurs within the cavity with the increasing of Ra. This is because of increasing convection within the cavity. It must be noticed that no significant change in streamlines occurs for increasing heat generation for higher values of Rayleigh number.

Fig. 4 describes the isotherms for three different values of Ra and heat generation parameter λ. For $\lambda = 0$ and $Ra = 10^3$, isotherms looks like parallel near the bottom wall of the cavity. This happens since the bottom surface is heated constantly and heated boundary layer is develops adjacent to the bottom wall. Due to the buoyancy effect, the hot fluid inside the boundary layer moves upward from the bottom left tip. With the increasing of heat generation, a boat shaped flow occurs as the rectangle block obstructs the flow. The temperature flows clockwise towards cold wall and becomes more clustered near the bottom wall of the cavity for increasing Ra. This happens because natural convection increases with the increasing Rayleigh number.

Fig. 5-6 represents the streamlines and isotherms for different values of Ra, height Hy and width Wx while

$\lambda = 10$ and $Pr = 0.71$. Fig. 5 (a)-(d) shows that for low convection two vortices formed within the cavity. The secondary cavity near the right inclined wall of the cavity vanishes with increasing natural convection. A small cell is formed at the left side of the rectangular block for the highest value of Ra. For fig. 5 (d), the cell takes place at the upper portion of the block due to decreasing the height of the block. In Fig. 5(b), for $Ra = 10^3, Wx = 0.3$ and $Hy = 0.1$, the size of the secondary vortex increase as it gets enough space for decreasing the width. In fig. 6 (a)-(d), the isotherms moves from lower wall to upper wall of the cavity and a clockwise recirculation flow occurs around the rectangular block. This occurs as the temperature differences between fluid and it always moves from hotter to colder.

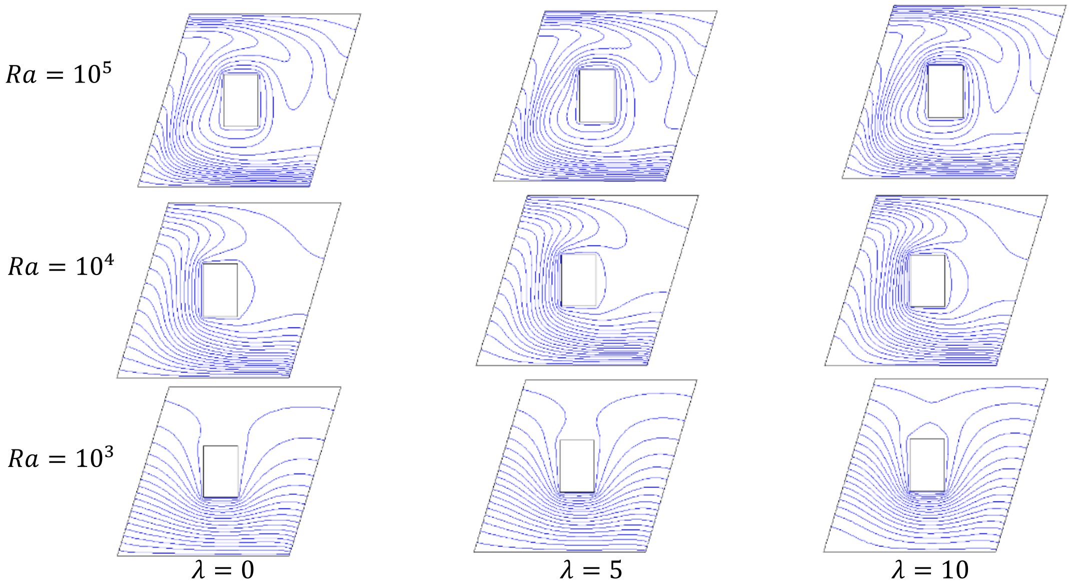

Fig.4. Isotherms for different values of Ra and heat generation parameter λ with $Pr = 0.71, Wx = 0.2$ and $Hy = 0.3$.

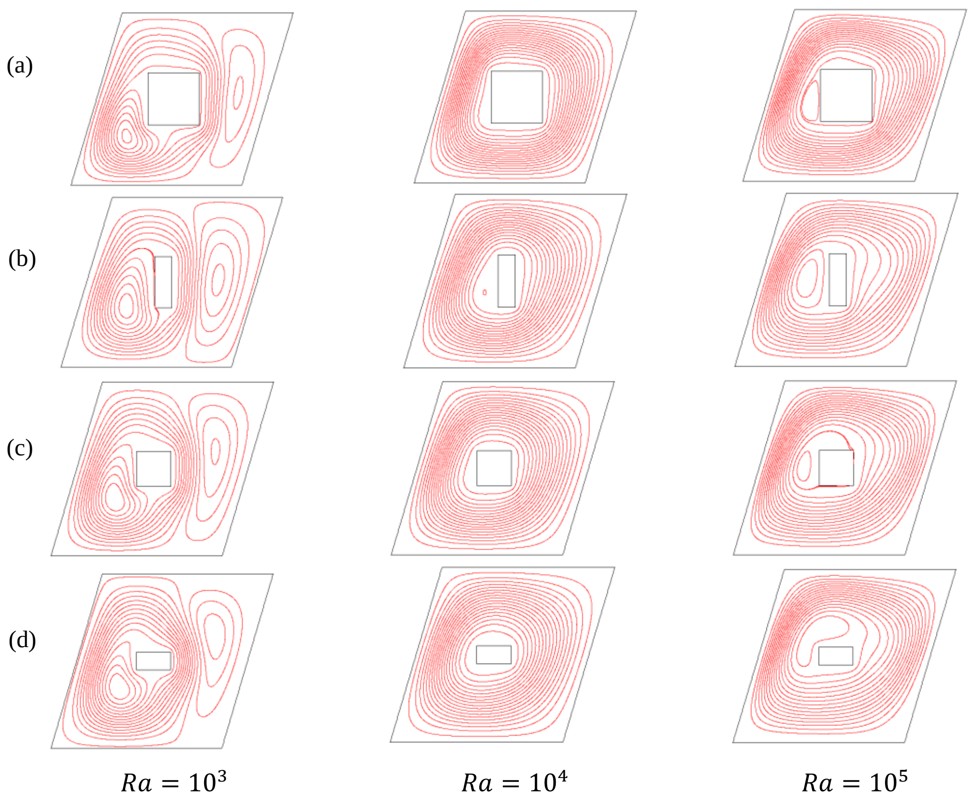

Fig.5. Streamlines for different values of Ra with $\lambda = 10$ and $Pr = 0.71; a)\ Wx = 0.3, Hy = 0.3, b)\ Wx = 0.1, Hy = 0.3, c)\ Wx = 0.2, Hy = 0.2, d)\ Wx = 0.2, Hy = 0.1$.

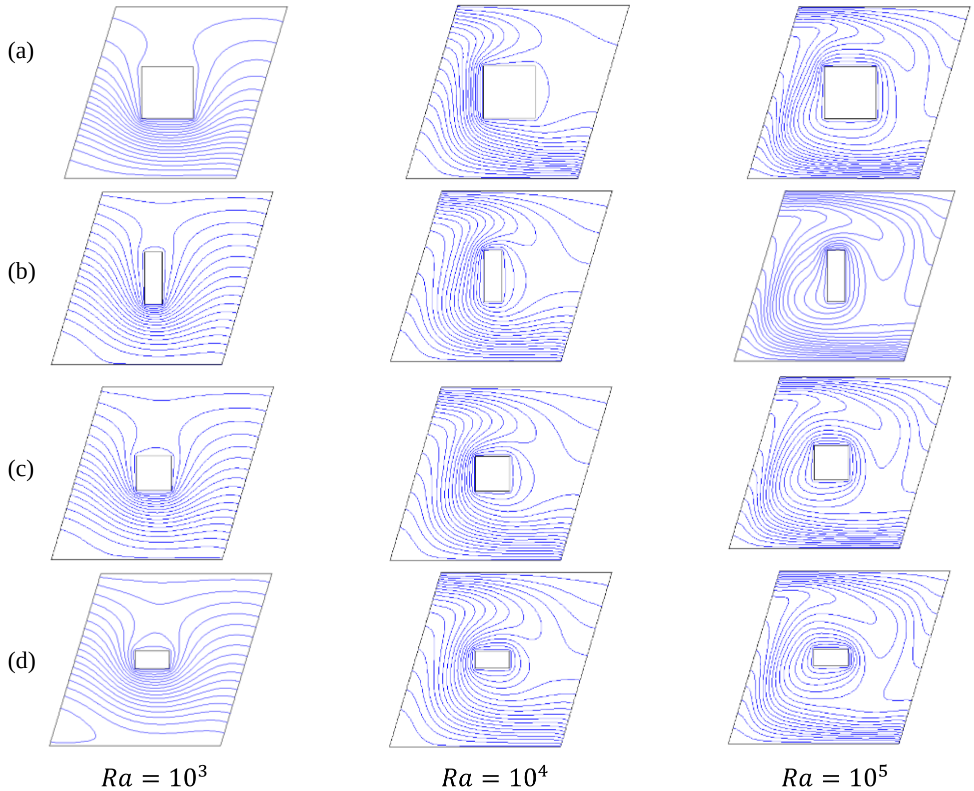

Fig.6. Isotherms for different values of Ra with $\lambda = 10$ and $Pr = 0.71$; $a)$ $Wx = 0.3, Hy = 0.3$, $b)$ $Wx = 0.1, Hy = 0.3$, $c)$ $Wx = 0.2, Hy = 0.2$, $d)$ $Wx = 0.2, Hy = 0.1$.

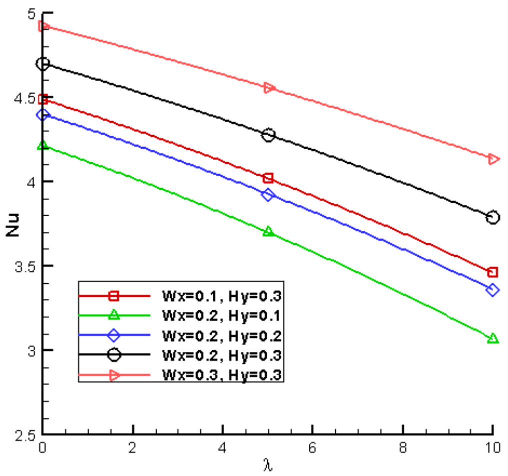

Fig.7. The variation of average Nusselt number Nu along the hot bottom wall for different heat generation parameter λ at $Wx = 0.1, Hy = 0.3$; $Wx = 0.2, Hy = 0.1$; $Wx = 0.2, Hy = 0.2$; $Wx = 0.2, Hy = 0.3$ and $Wx = 0.3, Hy = 0.3$. with $Ra = 10^5$.

Fig. 7 represents the graph as a function of average Nusselt number Nu and heat generation parameter λ for different values of $\boldsymbol{Wx}$ **and** $\boldsymbol{Hy}$ of the heated wall of the cavity. It can be noticed that the Nu is highest for $\boldsymbol{\lambda = 0}$ for all values of $\boldsymbol{Wx}$ **and** $\boldsymbol{Hy}$, as expected. This is due to the fact that, the heat generation mechanism creates a layer of hot fluid near the hot surface as a result the increasing rate of internal heat generation negates the heat transfer from the hot surface. The figure also shows that the $\boldsymbol{Nu}$ reduces with the increase of λ for any shaped of the rectangular block. The highest $\boldsymbol{Nu}$ is found for large square shaped of the block ($\boldsymbol{Wx = 0.3, Hy = 0.3}$) while lowest $\boldsymbol{Nu}$ presents for $\boldsymbol{Wx = 0.2}$ **and** $\boldsymbol{Hy = 0.1}$.

8. CONCOLUSION

In this study we conducted a numerical simulation to investigate the effect of rectangular block and heat generation on flow pattern and temperature field in a rhombic cavity. The outcomes of the existing analysis are as follows:

- The fluid flow and temperature field are strongly affected by the presence of the rectangular block that obstructs the flow and temperature fields.

- The heat generation effect is insignificant for high Rayleigh number.
- The average Nusselt number becomes significantly worse with the increase of heat generation and all shape of rectangular block.
- The rectangular block with dimension 0.3×0.3 in the absence of heat generation gives the highest heat transfer performance.
-

9. ACKNOWLEDGEMENT

This work is supported by the Department of Mathematics, Bangladesh University of Engineering and Technology and Department of Natural Science, Stamford University Bangladesh.

10. REFERENCES

[1] D. A. Nield, A. Bejan, "Convection in Porous Media", *second ed., Springer*, NY, 2006.

[2] D. B. Ingham, I. Pop, "Transport Phenomena in Porous Media II", *Pergamon*, 2005.

[3] S. Ostrach, "Natural convection in enclosures", *Adv. Heat Transfer*, vol. 8, pp. 161-227, 1972.

[4] Md. Mamun Molla, M. M. A. Sarker, "Natural convection flow in a square cavity with temperature dependent heat generation", *proceedings of 3rd BSME-ASME International Conference on Thermal Engineering, Bangladesh*

[5] Orhan Aydin, Ahmet U. Nal, Teoman Ayhan, "Natural convection in rectangular enclosures heated from one side and cooled from the ceiling", *International Journal of Heat and Mass Transfer*, vol. 42, pp. 2345-2355, 1999.

[6] Z.Y. Zhong, K.T. Yang, J.R. Lloyd, "Variable property effects in laminar natural convection in a square enclosure", *Journal of Heat Transfer*, vol. 107,pp. 133-138, 1985.

[7] M. Hasnaoui, E. Bilgen, P. Vasseur, "Natural convection heat transfer in rectangular cavities heated from below", *Journal of Thermophysics and Heat Transfer*, vol. 6, pp. 255-264, 1992

[8] M.M. Ganzarolli, L.F. Milanez, "Natural convection in rectangular enclosures heated from below and symmetrically cooled from the sides", *International Journal of Heat and Mass Transfer*, vol. 38, pp. 1063-1073, 1995.

[9] M. A. Hossain, M. Wilson, "Natural convection flow in a fluid-saturated porous medium enclosed by non-isothermal walls with heat generation", *Int. J. Therm. Sci*,. vol. 41, pp. 447-454, 2002.

[10] T.H. Chen, L.Y. Chen, "Study of buoyancy-induced flows subjected to partially heated sources on the left and bottom walls in a square enclosure", International Journal of Thermal Sciences, vol. 46,pp.1219–1231, 2007.

[11] Raju Chowdhury, Salma Parvin, Md. Abdul Hakim Khan, Ali J. Chamkha, "Effect of Magnetic Field and Heat Generation on Free Convection in a Porous Media Filled Equilateral Triangular Cavity", vol. 7, PP.49-61, 2015

[12] M. M. Rahman, M. A. H. Mamun, M. M. Billah and R. Saidur, "Natural convection flow in a square cavity with internal heat generation and a flush mounted heater on a side wall", Journal of Naval Architecture and Marine Engineering, vol. 7, pp. 37-50, 2007.

[13] R. Anandalakshmi, Tanmay Basak, "Heat flow visualization analysis on natural convection in rhombic enclosures with isothermal hot side or bottom wall", *European Journal of Mechanics B/Fluids*, vol. 41, pp. 29–45, 2013.

[14] Moukalled F, Darwish M, "Natural-convection heat transfer in a porous rhombic annulus", Numerical Heat Transfer A – Application, vol. 58, pp.101–24, 2010.

[15] Y. Varol, H.F.Oztop, T. Yilmaz, "Two dimensional natural convection in a porous triangular enclosure with a square body", *Int. Commun.Heat Mass Transfer*, vol. 34, pp. 238-247, 2007.

[16] A. Amine, J.K. Platten, M. Hasnaoui, "Thermal convection around obstacles: the case of Sierpinski carpets", *Exp. Fluids*, vol. 36, pp. 717-727, 2004.

[17] M.Y. Ha, I.K. Kim, H.S. Yoon, K.S. Yoon, J.R. Lee, S. Balachandar, H.H. Chun, Two-dimensional and unsteady natural convection in a horizontal enclosure with a square body", *Numer. Heat Transf., A Appl.*, Vol. 41, pp. 183-210, 2002.

[18] R. Chowdhury, M. A. H. Khan, M. N A. Siddiki, "Natural Convection in Porous Triangular Enclosure with a Circular Obstacle in Presence of Heat Generation", *American Journal of Applied Mathematics*, vol. 3, No. 2, pp. 51-58, 2015.

[19] S. Parvin and R. Nasrin, "Analysis of the Flow and Heat Transfer Characteristics for MHD Free Convection in an Enclosure with a Heated Obstacle", *Nonlinear Analysis: Modelling and Control*, vol. 16, pp. 89-99, 2011.

11. NOMENCLATURE

Symbol	Meaning	Unit
C_p	Specific heat at constant pressure	$(Jkg^{-1}K^{-1})$
g	Gravitational acceleration	(ms^{-2})
h	Height of the rhombus	(m)
hy	Height of the rectangular body	(m)
k	Thermal conductivity of the media	$(Wm^{-1}K^{-1})$
p	Fluid pressure	(Pa)
T	Temperature	(°C)
u, v	Velocity in x, y –direction	(ms^{-1})
wx	Width of the rectangular body	(m)
x, y	Cartesian coordinates	(m)
α	Thermal diffusivity	(m^2s^{-1})
β	Thermal expansion coefficient of fluid	(K^{-1})
θ	Dimensionless temperature	
λ	Dimensionless heat generation parameter	
μ	Dynamic viscosity	(Pas^{-1})
ν	Kinematic viscosity	(m^2s^{-1})
ρ	Fluid density	(kgm^{-3})